DERIVATIVES (Chain Rule and Product Rule)

Remember the following:

a) when you simplify radicals work with fractional exponents
b) when you move an expression from the denominator to the numerator the sign on the exponent changes from a positive to a negative.
c) when you subtract one from a negative exponent the exponent gets smaller (larger negative value) ex: $x^{-3} \Rightarrow x^{-3-1} \Rightarrow x^{-4}$
d) when removing a common factor from an expression always look for the smallest common exponent

ex: $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} \Rightarrow x^{\frac{1}{3}}\left(x^{\frac{1}{3}} + 5\right)$ $\qquad$ $x^{-\frac{5}{8}} + x^{-\frac{3}{8}} + x \Rightarrow x^{-\frac{5}{8}}\left(1 + x^{\frac{2}{8}} + x^{\frac{13}{8}}\right)$

1. $f(x) = x^2 + 4x - 6$
$f'(x) = 2x + 4$

2. $f(x) = x^3 - 5x^2 + 4x + 11$
$f'(x) = 3x^2 - 10x + 4$

3. $f(x) = (x^6 - x^4 + 2x)$
$f'(x) = 6x^5 - 4x^3 + 2$

4. $f(x) == (x^{12} + x^7 - x^3 + 2)$
$f'(x) = 12x^{11} + 7x^6 - 3x^2$

5. $f(x) = (x + 1)^2$
$f'(x) = 2(x+1) \cdot 1$
$f'(x) = 2(x+1)$

6. $f(x) = (x-5)^4$
$f'(x) = 4(x-5)^3$

7. $f(x) = (4x+2)^5$
$f'(x) = 5(4x+2)^4 \cdot 4$
$f'(x) = 20(4x+2)^4$
$f'(x) = 40(2x+1)^4$

8. $f(x) = (9x+5)^7$
$f'(x) = 7(9x+5)^6 \cdot 9$
$f'(x) = 63(9x+5)^6$

9. $f(x) = (7x+3)^{\frac{1}{2}}$
$f'(x) = \frac{1}{2}(7x+3)^{-\frac{1}{2}} \cdot 7$
$f'(x) = \frac{7}{2(7x+3)^{\frac{1}{2}}}$

10. $f(x) = \left(x^2 + 3x\right)^{\frac{5}{3}}$
$f'(x) = \frac{5}{3}\left(x^2 + 3x\right)^{\frac{2}{3}} \cdot (2x+3)$

11. $f(x)=\left(7x^3+6x-5\right)^{-\frac{3}{7}}$

$f'(x)=\frac{-3}{7}\left(7x^3+6x-5\right)^{-\frac{10}{7}}\cdot(21x^2+6)$

$f'(x)=\frac{-9(7x^2+2)}{7\left(7x^3+6x-5\right)^{\frac{10}{7}}}$

12. $f(x)=\sqrt{x^3+5x}$

$f(x)=(x^3+5)^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}(x^3+5x)^{-\frac{1}{2}}\cdot(3x^2+5)$

$f'(x)=\frac{(3x^2+5)}{2(x^3+5x)^{\frac{1}{2}}}$

$f(x)=\sqrt[3]{5x^3+7x-4}$

$f(x)==(5x^3+7x-4)^{\frac{1}{3}}$

13. $f'(x)=\frac{1}{3}(5x^3+7x-4)^{\frac{-2}{3}}\cdot(15x^2+7)$

$f'(x)=\frac{(15x^2+7)}{3(5x^3+7x-4)^{\frac{2}{3}}}$

$f(x)=\sqrt[5]{(5x-3)^4}$

$f(x)=(5x-3)^{\frac{4}{5}}$

14. $f'(x)=\frac{4}{5}(5x-3)^{\frac{-1}{5}}\cdot 5$

$f'(x)=\frac{4}{(5x-3)^{\frac{1}{5}}}$

15. $f(x)=\sqrt[7]{\left(4x^2-5x+1\right)^2}$

$f(x)=\left(4x^2-5x+1\right)^{\frac{2}{7}}$

$f'(x)=\frac{2}{7}\left(4x^2-5x+1\right)^{\frac{-5}{7}}\cdot(8x-5)$

$f'(x)=\frac{2(8x-5)}{7\left(4x^2-5x+1\right)^{\frac{5}{7}}}$

16. $f(x)=\sqrt[3]{(x^2+3)^6}$

$f(x)=(x^2+3)^{\frac{6}{3}}=(x^2+3)^2$

$f'(x)=2(x^2+3)\cdot 2x$

$f'(x)=4x(x^2+3)$

$f(x)=(4x^3+3)^{-2}$

17. $f'(x)=-2(4x^3+3)^{-3}\cdot 12x^2$

$f'(x)=\frac{-24x^2}{(4x^3+3)^3}$

$f(x)=(5x^3-2x^2+3x)^{-3}$

$f'(x)=-3(5x^3-2x^2+3x)^{-4}\cdot(15x^2-4x+3)$

18. $f'(x)=\frac{-3(15x^2-4x+3)}{(5x^3-2x^2+3x)^4}$

$f'(x)=\frac{-3(15x^2-4x+3)}{x^4(5x^2-2x+3)^4}$

$f(x)=(2x^2+x-1)^{-2}$

19. $f'(x)=-2(2x^2+x-1)^{-3}\cdot(4x+1)$

$f'(x)=\frac{-2(4x+1)}{(2x^2+x-1)^3}$

$f(x)=(4x^4-3x^2+5)^{-6}$

20. $f'(x)=-6(4x^4-3x^2+5)^{-7}\cdot\left(16x^3-6x\right)$

$f'(x)=\frac{-6\cdot 2x\left(8x^2-3\right)}{(4x^4-3x^2+5)^7}=\frac{-12x\left(8x^2-3\right)}{(4x^4-3x^2+5)^7}$

$f(x) = x^{-3} + 2x^{-4} - 3x^{-1}$

21. $f'(x) = -3x^{-4} - 8x^{-5} + 3x^{-2}$

$f'(x) = x^{-5}\left(-3x - 8 + 3x^{3}\right)$

$f(x) = x^{-7} + 5x^{-3} - 9x$

$f'(x) = -7x^{-8} - 15x^{-4} - 9$

22. $f'(x) = x^{-8}\left(-7 - 15x^{4} - 9x^{8}\right)$

$f'(x) = \dfrac{\left(-7 - 15x^{4} - 9x^{8}\right)}{x^{8}}$

$f(x) = \dfrac{1}{(x-3)} = (x-3)^{-1}$

23. $f'(x) = -1(x-3)^{-2}$

$f'(x) = \dfrac{-1}{(x-3)^{2}}$

$f(x) = \dfrac{5}{(2x-7)} = 5(2x-7)^{-1}$

24. $f'(x) = 5 \cdot -1(2x-7)^{-2} \cdot 2$

$f'(x) = \dfrac{-10}{(2x-7)^{2}}$

$f(x) = \dfrac{9}{(4x-2)^{3}} = 9(4x-2)^{-3}$

25. $f'(x) = 9 \cdot -3(4x-2)^{-4} \cdot 4$

$f'(x) = \dfrac{-108}{(4x-2)^{4}} = \dfrac{-108}{2^{4}(2x-1)^{4}} = \dfrac{-27}{4(2x-1)^{4}}$

$f(x) = \dfrac{9}{\left(4x^{3} - 7x\right)^{2}} = 9\left(4x^{3} - 7x\right)^{-2}$

26. $f'(x) = 9 \cdot -2\left(4x^{3} - 7x\right)^{-3} \cdot (12x^{2} - 7)$

$f'(x) = -\dfrac{9(12x^{2} - 7)}{2x^{3}(4x^{2} - 7)^{3}}$

$f(x) = \dfrac{5}{\sqrt{4x-6}} = 5(4x-6)^{\frac{-1}{2}}$

27. $f'(x) = 5 \cdot -\dfrac{1}{2}(4x-6)^{\frac{-3}{2}} \cdot 4$

$f'(x) = -10(4x-6)^{\frac{-3}{2}}$

$f'(x) = \dfrac{-10}{(4x-6)^{\frac{3}{2}}} = \dfrac{-10}{2^{\frac{3}{2}}(2x-3)^{\frac{3}{2}}}$

$f(x) = \dfrac{7}{\sqrt[4]{3x^{2} - 7x + 1}} = 7\left(3x^{2} - 7x + 1\right)^{\frac{-1}{4}}$

28. $f'(x) = 7 \cdot -\dfrac{1}{4} \cdot \left(3x^{2} - 7x + 1\right)^{\frac{-5}{4}} \cdot (6x - 7)$

$f'(x) = \dfrac{-7(6x-7)}{4\left(3x^{2} - 7x + 1\right)^{\frac{5}{4}}}$

$f(x) = \dfrac{9}{\sqrt[5]{(4x-5)^{3}}} = 9(4x-5)^{\frac{-3}{5}}$

29. $f'(x) = 9 \cdot -\dfrac{3}{5}(4x-5)^{\frac{-8}{5}} \cdot 4$

$f'(x) = \dfrac{-108}{5(4x-5)^{\frac{8}{5}}}$

$f(x) = (2x+3)(4x-5) = 8x^{2} + 2x - 15$

30. $f'(x) = 16x + 2 = 2(8x+1)$